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CRACK DECELERATION AND ARREST PHENOMENA AT AN OBLIQUE BIMATERIAL INTERFACE

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Abstract—The method of caustics with high-speed photography was employed to study the dynamic behavior of a transverse crack in bimaterial plates, with slant interfaces. Both phases of the specimens were fabricated from epoxy-polymers by casting the one phase successively after the other along its oblique boundary.

The results have shown that the cracks propagate, in general, under mode-I conditions and that the interface acts as a decelerator for the propagating crack, which, after passing the obstacle of the interface region, accelerates. The observed decelerations and accelerations were always greater in the brittle phases than those of the ductile ones.

The influence of the interface region to the crack propagation velocities was found to be more intense, when the crack propagates from a more brittle to a more ductile phase, than in the opposite case.

The zone of interaction of the crack with the interface was wide in the case of the inclined interface, increasing with the angle between interface and the loading axis. In this zone the stress intensity factor at first increases, then decreases to a minimum value on the interface and again increases, as the crack enters in the second phase. The crack paths were always slightly curved with a discontinuous tangent at the interface.

INTRODUCTION

The applicability of fracture mechanics to crack propagation and failure in fibrous composites is still open to question[1]. Indeed, phenomena taking place when a fast crack leaves the matrix and enters in a fiber or the other way around are of great interest. These phenomena of interaction between a fast running crack and a bimaterial rigid interface can be analysed by means of fracture mechanics by using the relevant fracture energy for each phase and crack propagation mode, and taking, also, into consideration the constraint imposed on each phase by the other.

The problem of the order of the crack tip singularity and the distribution of stress in the vicinity of the tip of a stable crack, terminating perpendicularly to a planar interface between two isotropic half spaces with different elastic constants, has been theoretically approached first by Zak and Williams[2]. They found that the stress singularity is no longer $\lambda = -1/2$ and the angular distribution of stresses differs considerably from that of a crack tip embedded in a homogeneous medium. Cook and Erdogan[3] have faced the same problem using the Mellin transform and solving the related integral equation. Bogy[4], using again the Mellin transform, has solved the problem of the crack tip singularity of a stable crack terminating at any angle to a planar bimaterial interface. Finally, Theocaris[5] suggested a method to determine the order of the singularity at the crack tip touching the interface at any angle by the method of caustics.

All these papers treat the interface as ideal, assuming that the materials are perfectly bonded along this mathematical surface. Also, the analysis of the stress system is a static one and, in principle, it does not allow any prediction about the dynamic behavior of a fast-running crack crossing a real interface, which is not a mathematical surface separating two materials. Rather it is a third intermediate phase[8] with its own mechanical properties acting as a material discontinuity.

The problem of the dynamic interaction between a propagating crack and a real interface has not been faced until now theoretically, but experimentally and only for cracks crossing normally the interface. The first studies on this subject[6, 7] were conducted by means of dynamic photoelasticity and were essentially focussed on the variation of the stress intensity factor K in the different material phases.

The problem has been faced first in a global and extensive way with the method of caustics in a series of papers by Theocaris *et al.*[9–12]. The first investigation by Dally *et al.*[7] studied the fracture process of duplex specimens, which were composed of a brittle material as first phase (notched) and a ductile material as second phase. The two material phases were joined together with a rather thick and tough high-shear strength epoxy layer. The results showed that the K-factor decreases as the crack approaches with a constant velocity the adhesive layer between the two phases and the crack stops abruptly as it reaches this layer. After a crack arrest time of about 120 μs, during which the K-factor monotonically increases and reaches a value corresponding to the toughness of the adhesive joint, the crack reinitiates in the second phase of the specimen. Finally, the K-factor decreases during the crack propagation in phase II, as the crack decelerates, and increases again slightly during the second crack arrest in phase II.

Sereda *et al.*[6] have studied the interaction between a fast running crack and a bimaterial interface, using specimens composed of three layers of epoxy polymers. In this case, the phases of the specimens were cast on each other. The behavior of the stress intensity factor, in the case where the crack approaches the interface from the side of the phase with a higher modulus of elasticity ($E_1/E_2 = 4$), has been found by Sereda *et al.* to be different than that observed in the experiments by Dally *et al.* In this case, the K-factor increases continuously as the crack arrives in the close vicinity of the interface and until it crosses the interface. After crossing the interface it decreases and, subsequently, it begins to grow again.

The same behavior of the K-factor has been observed in the experiments by Theocaris *et al.*, so that the discrepancy in the results concerning the K-factor must be related to the different preparation of the specimens used in the experiments, i.e. to the existence of the intermediate adhesive layer in Dally's experiments. The specimens used in the experiments by Theocaris *et al.* were composed of two epoxy-polymer phases, which were cast together along their common interface.

The influence of both the rigid interface and the variation of the material characteristics of each phase, on the stress system around the running crack tip, as well as on the magnitude and the variation of the crack propagation velocities, has been studied systematically. The results of the investigation lead to the conclusion that the bimaterial interface plays the role of a *barrier* to the crack propagation.

In the present paper an investigation was undertaken for dynamic crack propagation in biphase plates having their interface inclined to the axis of load. The paper describes the experimental procedure and the observed results.

THE SPECIMENS AND THE EXPERIMENTAL PROCEDURE

The specimens used in the tests were bimaterial epoxy-polymer specimens with a length of 0.30 m, a width of 0.10 m and a thickness of 0.003 m. Every specimen was composed of two phases with a trapezoidal form (Fig. 1) and the interface was inclined to the direction of the load-axis with an angle ϑ of 30, 45 or 60°.

At the free boundary of phase I we had an initial transverse slit of length $a_0 = 0.02$ m, which had a maximum distance between its adjacent lips about 0.0003 m. The length of the specimens inside each grip of the loading machine was 0.025 m, so that the free length 2h (Fig. 1) of the specimens tested was always 0.25 m.

The specimen materials were prepared from a pure cold-setting epoxy prepolymer Epicote 828 (Shell Co.), polymerized by addition of 8% triethylenetetramine (TET) hardener per weight of the epoxy prepolymer. The plasticizer, added to percentages from 0 to 50% of the epoxy prepolymer in steps of 10%, was a polysulfide polymer Thiocol LP3. This was done in order to have in the two faces a thoroughly known[13] variation of the material constants (see Table 1).

The two phases were bonded together without any further adhesive layer by casting the material of phase II along the interface of the already prepared phase I. After consolidation of phase II, the bimaterial plate was treated thermally for a period of a week with a maximum temperature of 110°C and a temperature gradient up and down of 3 degrees per hour. The reason for this treatment was to ensure a total polymerisation and, furthermore, an interface with minimum shrinkage stresses, much lower than the singular components of stresses. Moreover, these stresses being almost constant along the interface did not add anything in the shape and size of caustics since the caustics depend on the gradient of the sum of stresses[14].

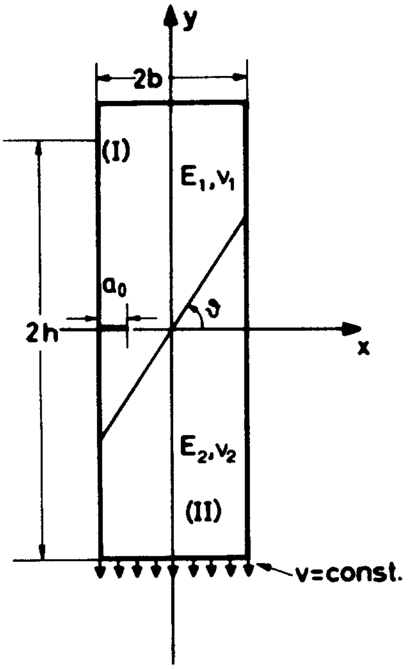


Fig. 1. Geometry of the cracked specimen.

Table 1. Mechanical constants, wave velocities and optical constants of epoxy polymers

Plasticizer c%	$E\times10^{-9}$ [Nm^{-2}]	ν	ρ [$kg\,m^{-3}$]	c_1 [ms^{-1}]	c_2 [ms^{-1}]	$c_t\times10^{9}$ [m^2N^{-1}]
0	3.30	0.338	1169	1785	1027	1.772
10	3.22	0.338	1179	1756	1010	2.017
20	3.00	0.340	1190	1688	970	2.097
30	2.50	0.358	1202	1545	875	2.197
40	1.82	0.430	1215	1356	724	3.138
50	1.30	0.480	1229	1172	598	3.781

For the dynamic fracture of the specimens we used a "Hydropulse" high-speed testing machine with electronic displacement control, maximum dynamic load of 40 kN and maximum displacement rate of 20 m/s. The applied tensile pulse was produced by the abrupt application of a displacement v at the lower transverse boundary of the specimen with a constant displacement rate and was recorded with a quarz-force transducer. As a measure of the dynamic load the average strain rate $\dot{\epsilon}$, defined as the rate of the applied constant displacement rate divided by the free length of the specimens, was considered. In our experiments we used strain rates in the interval between 1 and 5 s^{-1}.

The propagating crack was recorded by a Cranz–Schardin high-speed camera, disposing 24 sparks with a maximum frequency of 10^6 frames per second. The optical method of caustics has been chosen for the tests of interaction of a fast running crack with an inclined bimaterial interface, because it allows the closest observation of the stress singularity as compared with all the other experimental methods. For more details on the method of caustics and its application in fracture mechanics see Ref. [14].

EXPERIMENTAL RESULTS AND DISCUSSION

The series of tests, executed in bimaterial plates with slant interfaces, have indicated a clear similarity with the phenomena appearing in cracks propagating in bimaterial plates with longitudinal interfaces. However, important differences in the attitude of the propagating crack and the behavior of the materials of the phases were also detected.

A first and important observation is that the crack propagates in both phases under approximate mode-I conditions. This remark is based on the fact that the caustic senses the existence of mixed-mode deformation by its rotation about the crack tip[14]. Since such angular displacement of the caustic was not detected in the tests (see Fig. 2), it may be derived, that regionally the stress field around the running crack-tip was a mode-I field.

The overall characteristics of the dynamic fracture observed in bimaterial plates with slant interfaces may be summarized as follows:

(i) The crack propagation in phase I, containing the initial artificial crack, takes place with a variable velocity that increases continuously from its initiation value up to a maximum value. The maximum value occurs at some critical distance between the crack tip and the axis of the interface.

(ii) After passing this maximum, the propagating crack decelerates continuously to a minimum value of velocity attained when the crack tip reaches the interface. This minimum value, in some tests, becomes equal to zero; i.e. the crack is arrested.

(iii) After the crack crosses the interface region, it starts to accelerate again up to some maximum value of velocity. This second maximum value is attained close to the interface region.

(iv) When the crack passes the maximum value of velocity it decelerates smoothly and tends to some constant velocity, which depends upon the plate configuration and the applied strain rate.

(v) Finally, when the crack approaches the opposite longitudinal boundary of the plate it decelerates or accelerates smoothly depending on the sequence of phases brittle–ductile or ductile–brittle respectively.

Then, the slant interface of the plate acts as a *decelerator* of the propagating crack and, in some cases as a *crack-arrester*. However, its influence on the mode of propagation of the crack is not so intense, as it is the case with normal interfaces, where the crack is always arrested for a time between 5 and 50 μs, depending of the properties of the bimaterial plate[9, 11]. The difference between normal and slant interfaces is only quantitative.

This difference is due to the fact that the influence of the normal interface on the behavior of the propagating crack is concentrated into a restricted zone that extends from the tip of the crack to the normal interface. In contrast, the influence of the slant interface starts far away from the point of the final meeting of the crack with the interface. Accordingly, it has been observed that, a decrease of angle ϑ has as a result to elongate and smoothen the variation of the crack velocity.

Figure 2 presents a series of photographs, which indicate the process of propagation of a crack in a bimaterial plate, whose phase I is more ductile than phase II and whose interface subtends an angle $\vartheta = 60°$ with the transverse axis of the plate. The plate was submitted to a stress pulse with a strain rate $\dot{\epsilon} = 4\,\text{sec}^{-1}$. Phase I in this specimen contained 30% of plasticizer to the weight of the epoxy polymer, whereas phase II only 10% of the same plasticizer.

Photos 2.1–2.5 indicate that the crack tip is in phase I, whereas photos 2.6–2.10 show the crack tip in phase II.

Figure 3 shows a series of plots of the crack velocity v_c versus the instantaneous crack length a for a series of tests indicating the variation of the crack velocity along the plate for different combinations of phases in the bimaterial plate. The trends of all these curves may be classified into two groups, the one corresponding to a brittle-ductile succession of phases in the plate (Fig. 3a), and the other to the inverse one (Fig. 3b). The crack-tip velocities were calculated accurately from the position of each caustic, as this was explained previously in Ref.[14], and by using the interpolation method of a second degree Lagrange polynomial.

The diagram in Fig. 4 indicates the variation of the crack-tip velocity in terms of the instantaneous crack length a, where the already described variations of crack velocity are apparent.

Concerning the smooth transition of the singular stress field accompanying the crack tip from phase I to phase II, we may observe in the photo 2.5 that although the crack-tip is very close to the interface, phase II remains insensitive to the existence of this singular stress field. On the contrary, photo 2.6, where the crack-tip is almost touching the interface, reminds the static case of a stable crack. The corresponding caustic remains unchanged in shape and only changes in size, since it corresponds now to a singular stress field with a stress singularity $r^{-0.485}$ instead of $r^{-0.5}$. The last may be evaluated from Ref.[5].

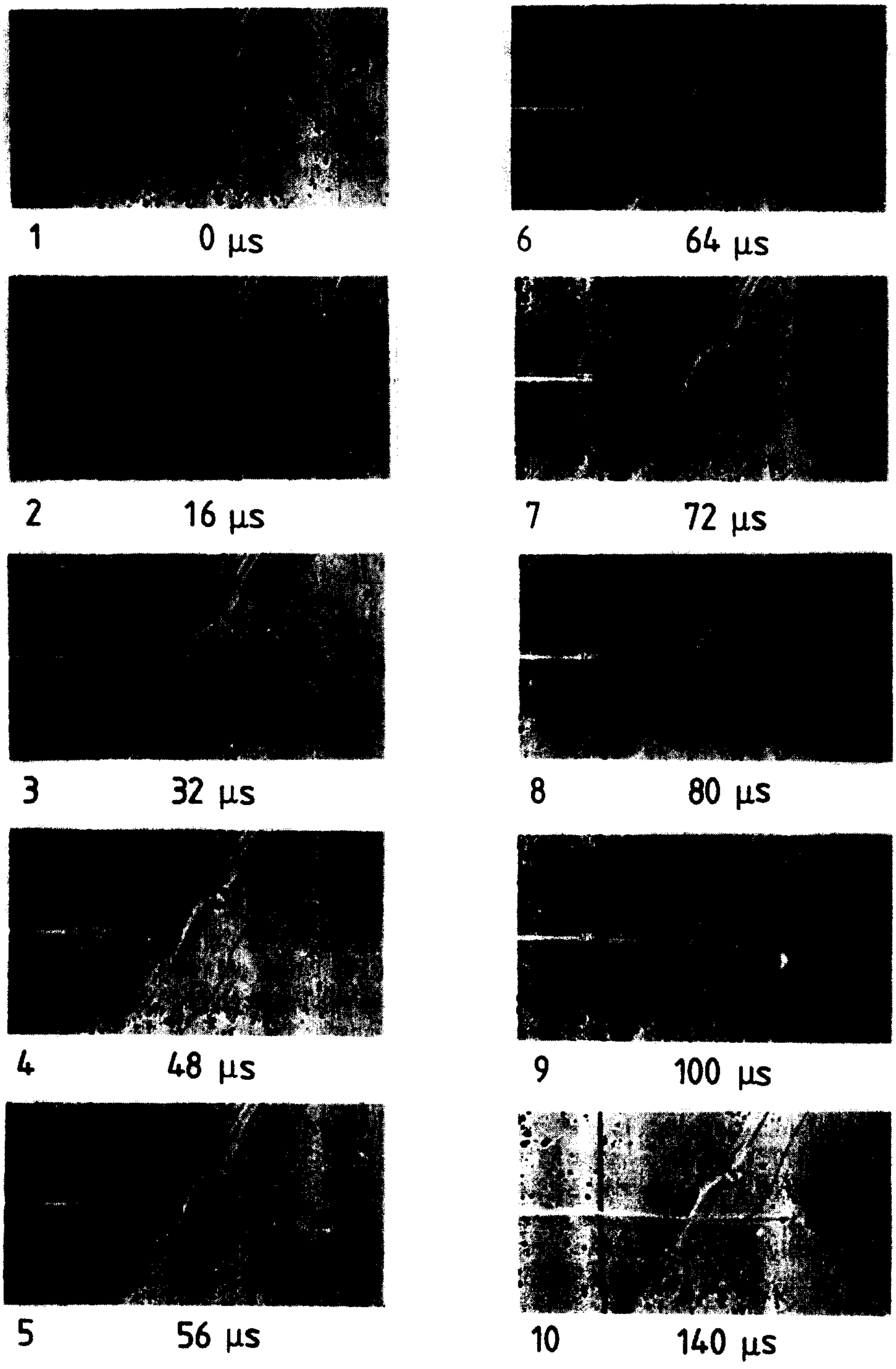
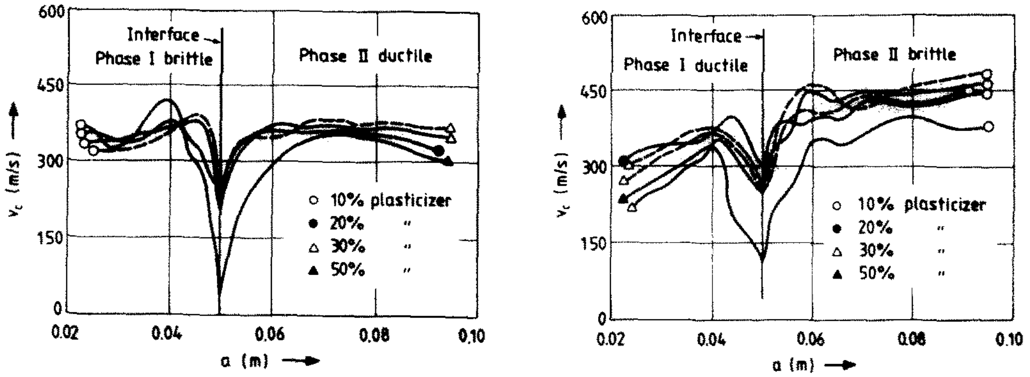


Fig. 2. Series of photographs showing crack propagation in a bimaterial specimen (phase I 30%, phase II 10% plasticized).



(a) (b)

Fig. 3. Variation of crack velocity v_c vs crack length a: (a) brittle–ductile succession of phases, full lines for $\vartheta = 45°$, dashed lines for $\vartheta = 60°$; (b) ductile–brittle succession of phases, dashed lines for $\vartheta = 45°$, full lines for $\vartheta = 60°$.

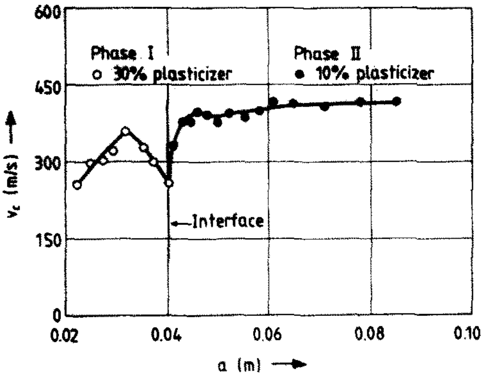


Fig. 4. Variation of crack velocity v_c vs crack length a for a specimen with $\vartheta = 60°$ and phase I 30%, phase II 10% plasticized.

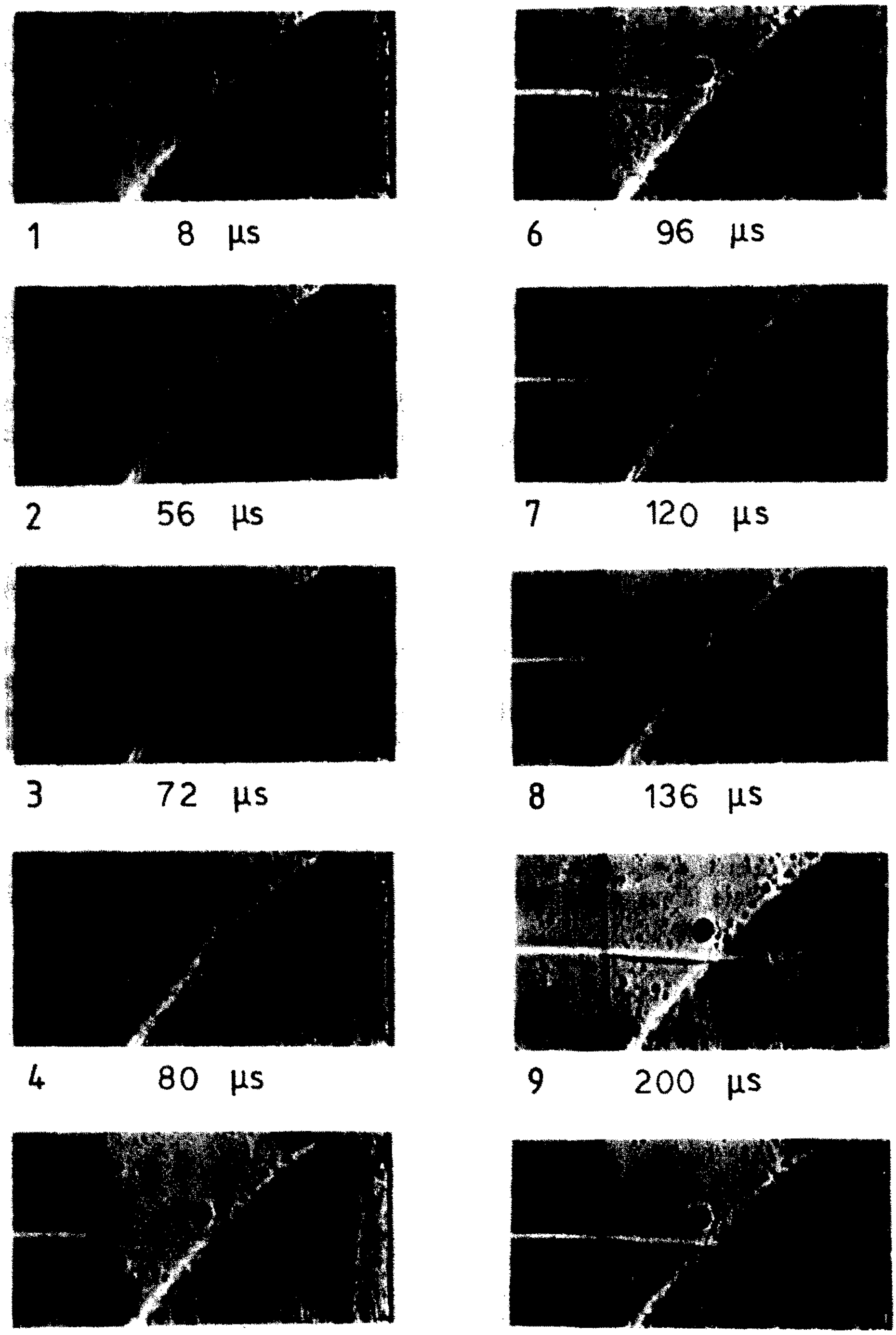
It should be mentioned here that the dynamic caustics differ in general from their static shapes and sizes[15]. However, it has been established[15], that for propagation velocities v_c not overpassing the $0.4c_1$, where c_1 is the longitudinal wave velocity of the plate (see Table 1), the shapes of dynamic caustics are essentially those of the static ones, while their size is slightly reduced. In our experiments v_c remained always smaller than $0.3c_1$, so that the shapes of dynamic caustics were not distorted and only their size was slightly reduced.

It may be deduced from the sequence of photos of Fig. 2 that, only at a restricted region in the close vicinity of the interface and when the crack tip still lies in phase I, there are shapes of caustics different from the typical almost circular caustics observed in static tests. These differences become more important for the cases of crack arrest, as it was already explained in Refs.[9, 11] for normal interfaces.

Figure 5 indicates such a case of crack arrest at the interface. The bimaterial plate was of the brittle–ductile type (10% (phase I)/50% (phase II) plasticizer added) and the angle $\vartheta = 45°$, while the applied strain-rate was $\dot{\epsilon} = 2\,\text{sec}^{-1}$. The sequence of photos 5.1–5.4 corresponds to cracks in phase I, whereas photos 5.5 and 5.6 correspond to the period of crack arrest at the interface. Finally, photos 5.7–5.10 correspond to cracks in phase II. We may observe that, when the crack tip lies at the vicinity of the interface, the caustic progressively disappears submerging in the interface zone, while its remaining part does not change shape. In photos 5.3–5.6 phase II remains unchanged and seems that it does not feel the approaching singular stress field of the crack. Moreover, there is an almost complete disappearance of the stress field in photos 5.5 and 5.6, when the crack is arrested. Only after a time lapse of 30 μs from the first disappearance of the caustic, a new caustic is starting to emerge in phase II. This caustic corresponds with a good approximation to a steady crack creating a singular stress field with a real stress singularity of the order $r^{-0.635}$, as this is evaluated according Ref.[5].

Finally, the crack, after its initiation in phase II, starts to propagate again under the influence of the external tensile pulse, which is quite long to cover the whole procedure up to the complete failure of the plate.

The disappearance of the caustic, which indicates the neutralization of the stress field around the crack tip, when the crack arrives at the discontinuity of the interface, may be explained by the rapid absorption of the kinetic and strain-energies stored at the propagating


1 8 μs

2 56 μs

3 72 μs

4 80 μs

5 88 μs

6 96 μs

7 120 μs

8 136 μs

9 200 μs

10 248 μs

Fig. 5. Crack-arrest at the interface (phase I 10%, phase II 50% plasticized).

crack-tip. Only after a time-period, during which the external load is still applied, strain energy is again stored at the crack-tip which progressively increases up to some limiting value, sufficient for an initiation and propagation of crack in phase II.

Moreover, in the case of crack arrest at the interface, a detailed examination of the interface zone has shown that the surface deformations at the vicinity of the interface are considerable, creating an important obstacle to the propagation of the singular stress-field around the crack-tip.

Experiments executed with bimaterial plates, whose lateral phases were polished after casting and curing and before testing, showed that the obstacle of the interface is considerably reduced, and there is a rather smooth transition of the crack from phase I to phase II. These results allow the hypothesis that an important factor affecting the mode of transition of the propagating crack from phase I to phase II is the quality of the material discontinuity at the zone of interface. The last depends to a large extent on the surface and structural anomalies of the interphase layer surrounding the interface. The influence of the quality of this interlayer zone was thoroughly studied in the series of papers [9–12].

We now consider in more details the mode of variation of the crack velocity in the plates. Figure 6 presents the variation of the crack velocity v_c in terms of the instantaneous crack length a in a typical experiment, where phase I was 30% plasticized (ductile phase) and phase II 10% plasticized (brittle phase). The angle of inclination of the interface was $\vartheta = 45°$ and the applied strain rate was $\dot{\epsilon} = 4\,\text{sec}^{-1}$. This experiment, therefore, is representative of all the characteristic parameters of the experiment of Fig. 4 with the only difference that angle ϑ is now $\vartheta = 45°$ instead of $\vartheta = 60°$.

We observe, that the maximum value of the crack velocity for both tests was the same and equal to $v_c = 357$ m/s. However, this maximum was attained at different places for different angles ϑ. Thus, for $\vartheta = 60°$ the distance between interface and position of maximum v_c was $d_{\max} = 0.008$ m, for $\vartheta = 45°$ this distance increased to $d_{\max} = 0.010$ m, indicating the influence of inclination of the interface to the behavior of the crack in phase I.

After this maximum, the crack was always decelerating and its velocity attained a minimum, lying always on the interface. The values of decelerations measured in all similar tests were about the value $b_a = 3 \times 10^6\,\text{ms}^{-2}$.

The crack transferred to phase II was accelerating again rapidly, with average accelerations of the order of $b_a = 6 \times 10^6\,\text{ms}^{-2}$, until its velocity reached again a maximum value. This second maximum was always about 10% larger than the respective maximum in phase I. Finally the maximum of velocity was followed by an almost constant velocity zone, until the influence of the opposite end of the bimaterial plate interferes, creating an insignificant increase of velocity near the boundary.

Figure 7 presents the variation of the crack-velocity v_c in terms of the instantaneous crack length for an inverse composition of the plate, that is, when phase I was brittle (10% plasticized) and phase II was ductile (20% plasticized). Here, angle ϑ was 45° and the applied strain rate was $\dot{\epsilon} = 2.4\,\text{sec}^{-1}$.

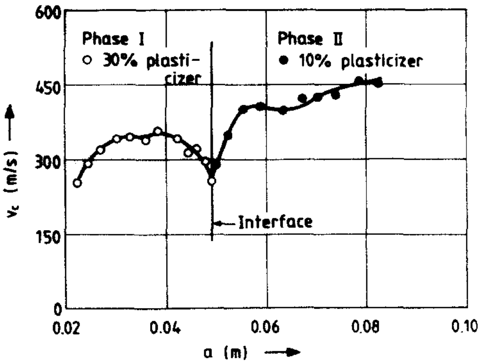


Fig. 6. Variation of crack velocity v_c vs crack length a for a specimen with $\vartheta = 45°$ and phase I 30%, phase II 10% plasticized.

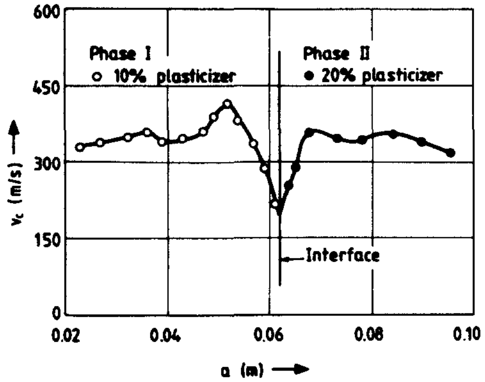


Fig. 7. Variation of crack velocity v_c vs crack length a for a specimen with $\vartheta = 45°$ and phase I 10%, phase II 20% plasticized.

In the case of this sequence of the phases in the plate the crack propagated in the brittle phase with almost constant velocity and an increase of v_c near the interface appeared corresponding to a maximum velocity of the order $v_c = 425\ \mathrm{ms}^{-1}$. After this maximum, the crack always decelerated, reaching a minimum value at the interface. The values of decelerations measured in this case were of the order of $b_a = 6 \times 10^6\ \mathrm{ms}^{-2}$, and the minimum values of the velocities were by 20–30% smaller than the respective minimum velocities at the interface for the ductile–brittle case.

The average accelerations of the crack-tip in the ductile phase were $b_a = 2$ to $3 \times 10^6\ \mathrm{ms}^{-2}$, and the second maximum of the velocity was smaller than the respective maximum in phase I. Finally the maximum of velocity in phase II was followed by an almost constant velocity zone with decreasing tendency near the boundary.

The rather complicated behavior of the propagation velocity was mainly due to two factors. The first factor was the influence of the actual interphase region between the two phases on the mode of propagation of the crack. This interphase is constituted of two strips on both sides of the common boundary of the constitutive phases of the specimen with different mechanical properties than the materials of the phases [8].

The second factor was that the crack in the bimaterial plates with slant interfaces was always propagating in a non-homogeneous dynamic stress field. Indeed, the crack, whose initial part was lying always in phase I, was propagating under the influence of a stress field created by the tensile pulse, applied at the bottom of the bimaterial plate, after it had traversed the interface.

Along this interface the pulse was submitted to reflections in phase II and to refractions in phase I and, therefore, the stress pulse arriving at the crack-tip was a compound pulse constituted of a longitudinal component and a transverse component. Because of the difference in the angles of refraction and reflection and of the difference in velocities between longitudinal and transverse waves in plates, which are given in Table 1, we may accept that the stress field engendering the propagation of the crack was strongly inhomogeneous.

In order to have a simple image of the stress field in which the crack was propagating, we calculated under the following simplifying assumptions the tensile $\sigma_y(x)$-stress in the bimaterial plate.

(i) The plate was submitted to a constant displacement v at its lower transverse boundary.

(ii) The plate was composed of independent bimaterial fibers parallel to the Oy-axis (Fig. 1) with continuously varying lengths in phases I and II, depending on the position of the fiber in the plate. With these assumptions it is valid that:

$$\sigma_y(x) = \frac{vE_I E_{II}}{h(E_I + E_{II}) + (E_I - E_{II})mx}, \tag{1}$$

where h is the half length of the plate, E_I and E_{II} are the moduli of elasticity of the phases I and II and $m = \tan\vartheta$.

Based on this simplified model, we may accept that the crack was propagating in a stress field which is variable, according to a law similar to the one expressed by relation (1). Thus, the crack was propagating in an increasing tensile stress field, when phase I was ductile ($E_I < E_{II}$), whereas it was propagating in a decreasing tensile field when phase I was a brittle one ($E_I > E_{II}$).

Finally, because of the small length of the trajectory of the crack relatively to its initial length, the crack propagation velocities in phase I, as well as in phase II, were always below the limiting values of velocities corresponding to the applied stress pulses and the respective geometries of the plates. These limiting values may be found from the approximate relationship[16]:

$$c_T \simeq 0.4(E/\rho)^{1/2}, \tag{2}$$

where E and ρ are the modulus of elasticity and the density of the homogeneous cracked plate and they are for unplasticized epoxy polymers $c_{T_0} = 670\,\mathrm{ms}^{-1}$ and for 50% plasticized epoxy polymers $c_{T_{50}} = 410\,\mathrm{ms}^{-1}$.

In conclusion, we may state that the variation of the crack velocity, which showed an increasing tendency in the case of a ductile-brittle propagation sequence, was the result of the superposition of two facts. Firstly, the crack was propagating with much lower velocities than the limiting velocities of the materials, and secondly, it was propagating in a continuously increasing tensile stress field. For the inverse case of a brittle-ductile sequence of propagation the two above-mentioned facts counteracted, resulting to an almost constant velocity.

Concerning the crack paths in the plates, it may be observed that in phase I the paths of the cracks were curved, whereas in phase II the cracks were propagating in straight lines normal to the direction of loading. There was also observed a difference in paths, depending on the sequence of phases. Thus, whereas in the brittle–ductile sequence the crack path was approaching the interface tangentially, as if it was repulsed by it, in the case of a ductile–brittle sequence the crack path was approaching normally the interface, as if it was attracted by it. Figures 8(a) and (b) present such typical crack paths. Figure 8(a) corresponded to a brittle-ductile sequence, whereas Fig. 8(b) to a ductile–brittle one.

In order to study the variation of the instantaneous values of the stress intensity factor, we assumed first that only the K_I-component was operative, according to the previously mentioned fact, that the crack propagates in both phases under approximate mode-I conditions. Furthermore, assuming that the order of singularity all over the plate was $r^{-(1/2)}$, the values of K_I-factor were determined according to the method of caustics[14].

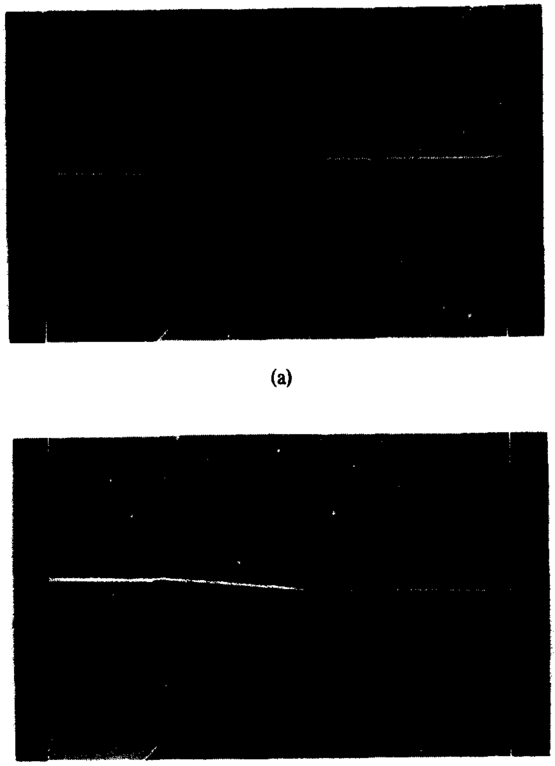
For the evaluation of K_I-stress intensity factor in the zone of influence of the interface, use was made of the approximate evaluation of K_I, as it has been developed in Ref.[11] for cracks approaching normally the interface. This is permissible, since, for the angles subtended by the crack-axis and the interface and the materials used in the bimaterial plate, the stress singularity at the crack tip remains always real and very close to its value $r^{-(1/2)}$ for the isotropic material[5].

The variation of K_I was similar with the variation of the crack propagation velocity, and it was differentiated according to the sequence of the phases in the plate.

In the case when the crack was propagating from the ductile to the brittle phase, K_I was diminishing at the beginning of the propagation and passed through a minimum value before any deceleration of the crack. This factor was afterwards increasing progressively up to a maximum value in phase I at a region closely to the interface. The maximum value was 20–30% higher than the average value for K_I. The passage of the crack through the interface was related with an abrupt reduction in the value of K_I, which was by 5–10% lower than its initial value.

In the brittle phase II, K_I was oscillating at the beginning and then it was continuously increasing as the crack velocity was increasing. The variation of K_I with the crack length is shown in Fig. 9 for the case where phase I is ductile (30% plasticizer) and phase II is brittle (10% plasticizer) and for angle $\vartheta = 60°$.

In the case when the crack was propagating from the brittle to the ductile phase, the factor K_I presented at the beginning an unstable behavior with oscillations and, afterwards, it stabilized to a continuous increase up to the neighborhood of the interface. As the crack was



(a)

(b)

Fig. 8. The paths of a crack in two specimens with $\vartheta = 45°$. (a) Phase I 10%, phase II 30% plasticized. (b) Phase I 30%, phase II 10% plasticized.

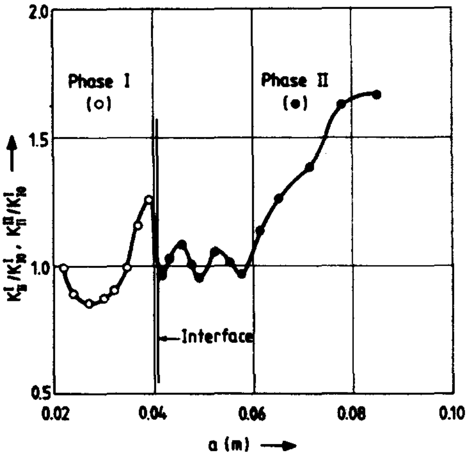


Fig. 9. The variation of the stress intensity factor, normalized at each phase to its initial value at the initiation of the crack in phase I (K_{I0}^I), as a function of the crack length a, in the case of a ductile–brittle succession (phase I 30%, phase II 10% plasticized).

approaching the interface and was decelerating, the K_I-factor increased abruptly to a maximum of about 40% above its initial value and then decreased rapidly to a 10% lower value than the average value. As soon as the crack crossed the interface, K_I increased rapidly to a maximum of about 120% of its initial value in phase I and, afterwards, it started to diminish, before the crack velocity attained its maximum.

Figure 10 shows the variation of K_I versus the crack length a for the case of a brittle phase I (10% plasticizer) and a ductile phase II (30% plasticizer) and for angle $\vartheta = 45°$.

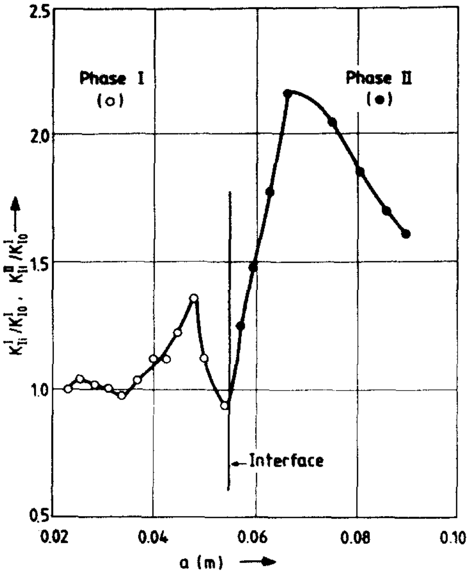


Fig. 10. The variation of the stress intensity factor, normalized at each phase to its initial value at the initiation of the crack in phase I (K^{I}_{I0}), as a function of the crack length a, in the case of a brittle–ductile succession (phase I 10%, phase II 30% plasticized).

Summarising the above referred results, we see that the variation of K_I is related to the variation of the crack velocity by a non-linear relation at the zone of the interface. Moreover, the K_I-factor presents an unstable behavior, which in the brittle phase of the plate is more intense than in the ductile phase.

CONCLUSIONS

From the experimental study of the dynamic crack propagation in bimaterial plates with a slant interface, we may conclude, that the interface behaves as a decelerator for the propagation of the crack. The influence of the interface on the propagating crack is related to the abrupt change of the mechanical properties of the plate at the interface. The actual interface cannot be considered as an ideal separating boundary between two different plates with different mechanical properties, but rather it is an interphase region. Moreover, the singular stress field around the crack tip, as the crack approaches the interface, is a complicated field and cannot be analyzed by static conceptions. Concerning the variation of the influence of the interface on the propagating crack with angle ϑ of obliqueness of the interface, we may conclude, that this influence decreases with decreasing ϑ and simultaneously it spreads out in a larger zone around the interface.

The propagation of the crack in both phases of the plate takes place under approximate mode-I conditions. The crack follows a slightly curved path, which adjusts continuously the crack-axis to the principal transverse axis of the stress field.

The variation of the crack propagation velocity during fracture of a bimaterial plate shows a complicated behavior with oscillating velocities due to consecutive decelerations and accelerations. In general, we may conclude that the accelerations and decelerations in the brittle phase are higher than the respective values in the ductile phase, and the influence of interface on the crack propagation velocity is also higher, when the crack propagates from the brittle to the ductile phase than in the inverse sequence.

Finally, the stress-intensity factor K_I shows an oscillating behavior, which is related to the similar behavior of the crack propagation velocity and is more intense in the brittle phase.

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REFERENCES

1. G. A. Cooper and M. R. Piggott, Cracking and fracture in composites. *Fracture* 1977, Vol. 1, ICF4 Waterloo, Canada. (Edited by D. M. R. Taplin) p. 557. Pergamon, Oxford (1977).
2. A. R. Zak and M. L. Williams, Crack point stress singularities at a bimaterial interface. *J. Appl. Mech.* **3**, 142 (1963).
3. T. S. Cook and F. Erdogan, Stresses in bonded materials with a crack perpendicular to the interface. *Int. J. Engng Sci.* **10**, 677 (1972).
4. D. D. Bogy, On the plane elastostatic problem of a loaded crack terminating at a material interface. *J. Appl. Mech.* **12**, 911 (1971).
5. P. S. Theocaris, Stress and displacement singularities near corners. *J. Appl. Math. and Phys.* (*ZAMP*) **26**, 77 (1975).
6. V. Ė. Sereda and V. M. Finkel, Interaction between fast crack and two-media interface. *Tanbov Inst. of Chem. Engng* UDC 192.45 translated from *Problemi Prochnosti* **12**, 24 (1977).
7. J. W. Dally and T. Kobayashi, Crack arrest in duplex specimens. *Int. J. Solids Structures* **14**, 121 (1978).
8. G. C. Papanicolaou, S. A. Paipetis and P. S. Theocaris, The concept of boundary interphase in composite mechanics. *Colloid and Polymer Sci.* **256**, 625 (1978).
9. P. S. Theocaris and J. Milios, Dynamic crack propagation in composites. *Int. J. Fract.* **16**, 31 (1980).
10. P. S. Theocaris and J. Milios, Crack propagation velocities in bi-phase plates under static and dynamic loading. *Engng Fracture Mech.* **13**, 599 (1979).
11. P. S. Theocaris and J. Milios, Crack-arrest at a bimaterial interface. *Int. J. Solids Structures* **17**, 217 (1981).
12. P. S. Theocaris and J. Milios, The process of the momentary arrest of a moving crack approaching a material discontinuity. *Int. J. Mech. Sci.* **23**, 423 (1981).
13. P. S. Theocaris and J. Prassianakis, Interrelation of mechanical and optical properties of plasticized epoxy polymers. *J. Appl. Polymer Sci.* **22**, 1725 (1978).
14. P. S. Theocaris, *Developments in Stress Analysis*, Vol. I (Edited by G. Holister) Chap. 2, p. 27 (1979). P. S. Theocaris, *Mechanics of Fracture* Vol. 7 (Edited by G. C. Sih) Chap. 3, p. 189 (1981).
15. P. S. Theocaris and G. A. Papadopoulos, Elastodynamic forms of caustics for running cracks under constant velocity. *Engng Fracture Mech.* **13**, 683 (1980).
16. E. N. Dulaney and E. F. Brace, Velocity behavior of a growing crack. *J. Appl. Phys.* **31**, 2233 (1960).